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Sub-barrier fission probability for a double-humped barrier

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Abstract. The interpretation of data on the decay of actinide nuclei by the fission process at excitation energies below the fission barrier is considered. It is shown that the intermediate structure (class-II levels), introduced into the fission strength function by the presence of a secondary well in the potential energy of deformation through the fission barrier, has a decisive effect in reducing the probability of fission decay at these energies. It is indicated how fission barrier parameters may be extracted from the data. Some actual examples given in this paper demonstrate the existence of an odd–even effect in fission barrier heights relative to the nuclear ground state.

1. Introduction

For present studies of nuclear structure at large deformations it is particularly important to have precise determinations of fission barrier parameters. Such determinations are complicated by the very factor that makes them so interesting, namely, the discovery, in actinide nuclei, of the double-humped barrier, which is explained by the theoretical work of Strutinsky (1967), and which provides a mechanism for the phenomena of spontaneously fissioning isomers (Flerov and Polikanov 1964, Lark *et al* 1969) and structure in the energy dependence of fission cross sections (Fubini *et al* 1968, Migneco and Theobald 1968). The barriers of many actinides, particularly odd-neutron, even-proton and doubly-odd nuclei, are higher than the neutron thresholds, and the presence of competition from neutron emission, comparable in strength to the fission decay of the compound nucleus, gives rise to a strongly increasing fission cross section close to the actual barrier height (which is taken as the greater of the two barrier maxima). For even compound nuclei, as well as odd-proton, even-neutron systems, the fission barrier is generally below the neutron threshold, and only the weak electromagnetic radiation process is in competition with fission. The actual strong fall in fission yield from such nuclei excited to below the neutron threshold by transfer reactions such as (d, p) and (t, p) can therefore occur some considerable energy below the barrier, and careful interpretation is necessary to deduce the true barrier height. It is the purpose of the present paper to draw attention to the effect on the fission probability of the detailed structure of the class-II compound levels associated with the secondary well of the double-humped barrier for such compound nuclei.

2. Calculation of sub-barrier fission probability

2.1. *Fission probability for uniform levels*

The transmission coefficient for fission is denoted by $T_F(=2\pi\Gamma_{(F)}/\bar{D}$ for narrow levels), and the transmission coefficient for all other decay processes from the compound nucleus is denoted by T'. Over an energy interval within which the levels have uniform properties the fission probability of the compound nucleus is

$$P_F = \frac{T_F}{T_F + T'}. \tag{1}$$

For a double-humped fission barrier, the fission transmission T_F can be expressed in terms of coefficients for transmission across the two peaks A and B (see figure 1) of the

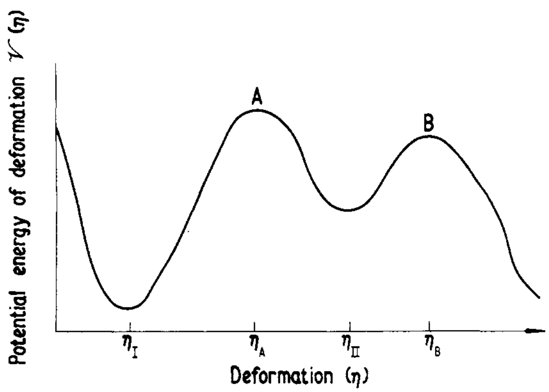

Figure 1. Schematic diagram of double-humped fission barrier as a function of deformation in the fission mode.

double-humped barrier separately; the coefficients are denoted by T_A, T_B and can be written for narrow class-II levels associated with the secondary well of the barrier as

$$T_A \simeq \frac{2\pi\bar{\Gamma}_{II(A)}}{\bar{D}_{II}} \tag{2a}$$

$$T_B \simeq \frac{2\pi\bar{\Gamma}_{II(B)}}{\bar{D}_{II}} \tag{2b}$$

in which $\bar{\Gamma}_{II(A)}$ is the average width for coupling of class-II states to the normally deformed class-I compound states, $\bar{\Gamma}_{II(B)}$ is the average fission width of the class-II states, and $\bar{D}_{II}$ is their average spacing. The fission transmission coefficient T_F for the compound state formed from coupling the class-I and class-II states is (Lynn 1968, Weigmann 1968)

$$T_F = \frac{\Gamma_{II(A)}\Gamma_{II(B)}}{(E_{II} - E)^2 + \frac{1}{4}(\Gamma_{II(A)} + \Gamma_{II(B)})^2} \tag{3}$$

in the neighbourhood of a typical class-II level (ignoring other class-II wings and background effects). The average fission transmission over the energy interval D_{II} is

$$\bar{T}_F = \frac{T_A T_B}{T_A + T_B} \tag{4}$$

On substitution of this into equation (1) the result

$$P_F = \frac{T_A T_B}{T_A T_B + T'(T_A + T_B)} \tag{5}$$

is achieved; this is a result that is commonly employed for analysis of fission probability measurements to find fission barrier parameters.

Equations (4) and (5) are here given implicitly in terms of the strong damping situation, in which $T_{A,B}$ have the Hill–Wheeler form

$$T_{A,B} = \left[1+\exp\left(\frac{2\pi(V_{A,B}-E)}{\hbar\omega_{A,B}}\right)\right]^{-1}, \tag{6}$$

in which $V_{A,B}$ are the barrier heights and $\hbar\omega_{A,B}$ their tunnelling parameters; the equations are more general, however, if $T_{A,B}$ are given the narrow interpretation of equations (2).

2.2. *Effect of class-II structure on the average fission probability*

At low energies, in particular below particle emission threshold, the transmission coefficient T' can be very small, being limited essentially to electromagnetic radiation processes, and can be exceeded greatly by the average fission coefficient $\bar{T}_F$ even well below the barrier height. In these circumstances the detailed class-II structure of T_F can be significant; if the bulk of the strength of T_F is concentrated in a narrow energy interval about the class-II level, there can be considerable energy intervals in which T' is of the order of or greater than T_F, and the average fission probability $\bar{P}_F$ may be considerably decreased in consequence. In the neighbourhood of a single class-II level the fission probability is

$$\begin{aligned} P_F &= \frac{T_A T_B}{(E_{II}-E)^2(4\pi^2 T'/D_{II}^2)+T_A T_B+\frac{1}{4}T'(T_A+T_B)^2} \\ &= \frac{D_{II}^2}{4\pi^2}\left(\frac{T_A T_B}{T'}\right)\left[(E_{II}-E)^2+\frac{D_{II}^2}{4\pi^2}\left(\frac{T_A T_B}{T'}+\frac{(T_A+T_B)^2}{4}\right)\right]^{-1} \end{aligned} \tag{7}$$

If this simple lorentzian form is averaged over D_{II}, the average fission probability

$$\bar{P}_F = \frac{1}{\pi}\frac{T_A T_B}{T'}\, Y\tan^{-1}(\pi Y) \tag{8}$$

where

$$Y = \left(\frac{T'}{T_A T_B+\frac{1}{4}T'(T_A+T_B)^2}\right)^{1/2}$$

is obtained.

Above the lower of the two barrier peaks the typical width of a class-II state approaches the class-II level spacing and the result of equation (8) will be very nearly that of the (structureless) equation (5). Likewise, a long way below the barrier peaks the fission transmission coefficient, as expressed in equation (3), will be almost everywhere much smaller than the radiation transmission T' (except when $\Gamma_{II(A)} \simeq \Gamma_{II(B)}$) and equations (8) and (5) will again give similar results. In the intermediate energy region, however, the result of equation (8) can be very different from that of (5), and this is shown numerically for some typical sets of barrier parameters in figure 2. These indicate that in some cases the height of the fission barrier can be over-estimated by $\frac{1}{2}$ MeV or more by the uncritical use of equation (5).

For uniform class-II levels the fission probability can be found exactly for the many-level situation in which the fission transmission coefficient is a sum over a long sequence of terms of the type in equation (3), the energy levels E_{II} being spaced uniformly at

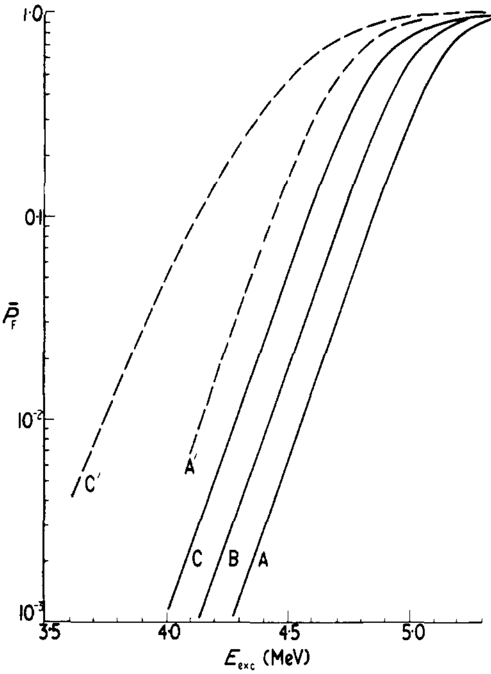

Figure 2. Fission probability below particle separation energies calculated from equations (8) and (5). Full curves A, B, C are calculated from the structure-based theory (equation (8)) with barrier parameters $V_A = 5{\cdot}9$ MeV, $\hbar\omega_A = 0{\cdot}9$ MeV, $\hbar\omega_B = 0{\cdot}6$ MeV, and $V_B = 5{\cdot}3$ MeV (curve A), 5·1 MeV (curve B) and 4·9 MeV (curve C). Broken curves A′ and C′ correspond to the parameters of A and C but are based on the less exact theory (equation (5)). The radiation transmission coefficient employed was $T_\gamma = 3{\cdot}33 \times 10^{-7}\, e^{E/0{\cdot}5575}$.

intervals of $\bar{D}_{II}$. The result is derived briefly in the appendix for the general case of partial damping. In this case the fission transmission coefficient is assumed to be composed of a direct and an indirect term

$$T_F = T_D + T_{Indr}$$

of which only the indirect term possesses the class-II compound structure. Under these assumptions we find

$$\bar{P}_F = \frac{T_D}{T_D + T'} + \left\{\left(1 + \frac{T_D}{T'}\right)\left[1 + \left(\frac{(T_D + T')(T_A + T_B)}{T_{Abs}T_B}\right)^2 + 2\,\frac{(T_D + T')(T_A + T_B)}{T_{Abs}T_B}\coth[\tfrac{1}{2}(T_A + T_B)]\right]^{1/2}\right\}^{-1} \tag{9a}$$

where T_{Abs} is the probability for absorption in the secondary well while attempting to penetrate the barrier. In the limit of *no damping* where $T_{Abs} = 0$ equation (9*a*) reduces to the expected expression

$$\bar{P}_F = \frac{T_D}{T_D + T'}. \tag{9b}$$

In the other limit of *complete damping*, where $T_{Abs} = T_A$ and $T_D = 0$ we find

$$\bar{P}_F = \left[1+\left(\frac{T'}{T_F}\right)^2+2\left(\frac{T'}{T_F}\right)\coth[\tfrac{1}{2}(T_A+T_B)]\right]^{-1/2} \qquad (9c)$$

This expression is illustrated in figure 3 for certain sets of barrier parameters and is compared with the expression (8). As expected, it is higher than (8) but only by about 10% at the most.

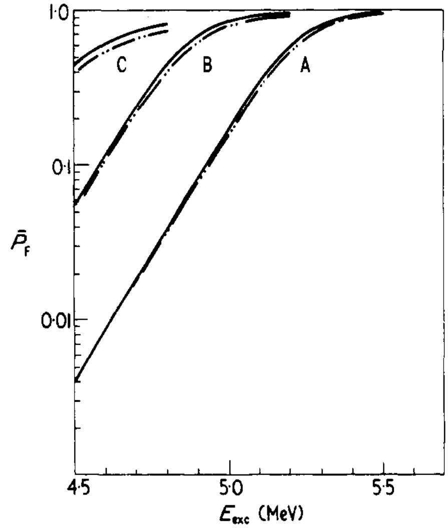

Figure 3. Fission probability calculated for single class-II level (chain curve) and many class-II level (full curve) approximations. Barrier parameters are $V_A = 5{\cdot}5$ MeV, $\hbar\omega_A = 0{\cdot}9$ MeV, $\hbar\omega_B = 0{\cdot}6$ MeV, and $V_B = 5{\cdot}5$ MeV (curves A), $V_B = 5{\cdot}0$ MeV (curves B) and $V_B = 4{\cdot}5$ MeV (curves C).

2.3. *Effect of statistical fluctuations in class-I levels*

It is well known that the true averages of decay probabilities over many levels cannot be written accurately in the form (1) if the partial widths for the decay processes in individual levels fluctuate statistically about the mean partial widths (Lane and Lynn 1957). In the present case where T is a radiative process the individual total radiative widths are expected to be essentially constant from level to level, whereas the fission widths (sub-barrier fission with only one transition state effective) are expected to have a Porter–Thomas (1956) distribution about their mean value as given by equation (3) (Bohr 1956) and the result of Lane and Lynn (1957) can be taken over, giving

$$P_F = \frac{T_F}{T_F+T_\gamma}\mathscr{S} \qquad (10a)$$

with

$$\mathscr{S} = \left(1+\frac{T_\gamma}{T_F}\right)\left\{1-\left(\frac{\pi T_\gamma}{2T_F}\right)^{1/2}\exp\left(\frac{T_\gamma}{2T_F}\right)\operatorname{erfc}\left\{\left(\frac{T_\gamma}{2T_F}\right)^{1/2}\right]\right\} \qquad (10b)$$

The numerical values of the fluctuation factor $\mathscr{S}$ lie between unity (for very large or small ratios of T_F to T_γ) and 0·68 (for $T_\gamma/T_F \simeq 0{\cdot}6$).

An accurate assessment of the effect of statistical fluctuations on the average fission probability over a single class-II level requires numerical integration of equation (10*b*) with equation (3) (or its generalization over many class-II levels) substituted for T_F, but a simple semi-quantitative discussion can be given. In the important sub-barrier regions of interest equation (7) will vary from very small values to a peak value of near unity in traversing a class-II level. The half-width of this lorentzian variation is measured across the points where $T_F = T_\gamma$. The effect of statistical fluctuations is to lower the fission probability curve to less than 70% of the uniform' model value in just this half-width region and thus reduce the effective width of the curve to about 70% of the uniform value. The average value of the fission probability is proportional to the half-width of the curve and is lowered in proportion. This approximate 70% factor is a lower limit on the reduction of the average fission probability since the tails of the curve are not lowered to the same extent.

2.4. *Statistical fluctuations of class-II levels*

For the model of pure class-II vibrations (no damping in the secondary well) the expressions so far derived are adequate; the class-II levels are not complicated compound levels in this case, but, rather, pure class-II vibrational levels, and their widths $\Gamma_{II(A)}$ and $\Gamma_{II(B)}$ can be deduced either from statistical theory expressions (Lynn 1969, Bjørnholm and Strutinsky 1969) or, more exactly, from numerical computations of transmission through a double-humped barrier (Back *et al* 1971, Wong and Bang 1969, Cramer and Nix 1970). If experimental resolution is insufficient to resolve the vibrational resonances, the analysis of fission probability should employ the strong coupling expressions, equation (8) or (9*c*), but if these resonances can be resolved then it is sufficient to use the simple expression (9*b*). When damping in the secondary well causes the mixing of these simple vibrational levels into more complicated class-II compound levels, the question arises of statistical fluctuations of the class-II widths about local mean values.

Such fluctuations are expected theoretically to be of the Porter–Thomas form (in sub-barrier fission with a single transmission state). Two possibilities must still be distinguished. One is that under certain kinds of simple damping the coupling and fission widths $\Gamma_{II(A)}$ and $\Gamma_{II(B)}$ may be correlated, while the other (which has certainly been observed experimentally, eg in slow neutron-induced fission of ^{240}Pu, Migneco and Theobald 1968) is that of no correlation between these widths.

Analytical expressions for the average fission probability incorporating such statistical fluctuations have not been sought. Instead, numerical averages of equation (8) with values of T_A and T_B chosen by selection using pseudo-random numbers from Porter–Thomas distributions have been carried out. In the sub-barrier energy range of interest the values of fission probability resulting from equation (8) are reduced by a factor that can be as low as approximately 0·6 in the case of uncorrelated widths $\Gamma_{II(A)}$ and $\Gamma_{II(B)}$, however, in the fully correlated case there is no reduction factor.

2.5. *Very low excitation energies: the perturbation regime*

At very low excitation energies the width of the lorentzian expression (3) becomes substantially less than the spacing of the class-I levels, $\bar{D}_I$. In this case the coupling of class-I and class-II levels is described adequately by first-order perturbation theory (Lynn 1968) which also implies correlation properties in the excitation and decay of the compound nucleus states. It has been assumed implicitly in the previous sections that the initial excitation of the compound states is independent of their decay mode. If the compound nucleus is initially excited through components of the wavefunction of class-I type (neutron excitation, transfer reactions of (d, p), (t, p) type etc) then in the perturbation regime there will be an anti-correlation between excitation probability and fission decay width. The perturbation expressions for the fission widths are, for the quasi-class-I states

$$\Gamma_{\lambda'(\mathrm{F})} \simeq \frac{H^2_{\mathrm{c}.\lambda_{\mathrm{I}}\lambda_{\mathrm{II}}}}{(E_{\lambda_{\mathrm{I}}}-E_{\lambda_{\mathrm{II}}})^2}\Gamma_{\lambda_{\mathrm{II}}(\mathrm{B})}, \tag{11}$$

and for the quasi-class-II state

$$\begin{aligned}\Gamma_{\lambda''(\mathrm{F})} &\simeq \Gamma_{\lambda_{\mathrm{II}}(\mathrm{B})}\left(1-\sum_{\lambda_{\mathrm{I}}}\frac{H^2_{\mathrm{c}.\lambda_{\mathrm{I}}\lambda_{\mathrm{II}}}}{(E_{\lambda_{\mathrm{I}}}-E_{\lambda_{\mathrm{II}}})^2}\right)\\ &\simeq \Gamma_{\lambda_{\mathrm{II}}(\mathrm{B})}\left(1-\frac{\pi^2\overline{H^2_{\mathrm{c}}}}{D_{\mathrm{I}}^2}\right)\end{aligned} \tag{12}$$

Widths for excitation of these states are proportional to

$$w_{\lambda'} \simeq 1-\frac{H^2_{\mathrm{c}.\lambda_{\mathrm{I}}\lambda_{\mathrm{II}}}}{(E_{\lambda_{\mathrm{I}}}-E_{\lambda_{\mathrm{II}}})^2} \tag{13}$$

for quasi class-I states, and

$$w_{\lambda''} \simeq \sum_{\lambda_{\mathrm{I}}}\frac{H^2_{\mathrm{c}.\lambda_{\mathrm{I}}\lambda_{\mathrm{II}}}}{(E_{\lambda_{\mathrm{I}}}-E_{\lambda_{\mathrm{II}}})^2} \simeq \frac{\pi^2\overline{H^2_{\mathrm{c}}}}{D_{\mathrm{I}}^2} \tag{14}$$

for quasi-class-II states. In addition to the prompt fission expressed by equations (11) and (12) it is usual that measurements of fission yield include delayed fission from low-lying class-II isomer states. Thus the radiation widths cascading to these states must be considered in calculating the fission probability. These widths are

$$\Gamma_{\lambda'(\gamma\to\mathrm{II})} \simeq \frac{H^2_{\mathrm{c}.\lambda_{\mathrm{I}}\lambda_{\mathrm{II}}}}{(E_{\lambda_{\mathrm{I}}}-E_{\lambda_{\mathrm{II}}})^2}\Gamma_{\lambda_{\mathrm{II}}(\gamma\to\mathrm{II})} \tag{15}$$

$$\Gamma_{\lambda''(\gamma\to\mathrm{II})} \simeq \Gamma_{\lambda_{\mathrm{II}}(\gamma\to\mathrm{II})}\left(1-\frac{\pi^2\overline{H^2_{\mathrm{c}}}}{D_{\mathrm{I}}^2}\right) \tag{16}$$

Of the radiation through class-II states given by these widths only a certain fraction will result in fission while the remainder will follow a radiative cascade chain to the normal ground state of the compound nucleus. The branching ratio for delayed fission is denoted by $b_{\mathrm{II}\to\mathrm{F}}$.

The probability of fission (prompt plus delayed) is given by

$$P_{\mathrm{F}} = \sum_{\lambda'}\frac{w_{\lambda'}(\Gamma_{\lambda'(\mathrm{F})}+b_{\mathrm{II}\to\mathrm{F}}\Gamma_{\lambda'(\gamma\to\mathrm{II})})}{w_{\lambda'}\Gamma_{\lambda_{\mathrm{I}}(\gamma\to\mathrm{I})}+\Gamma_{\lambda'(\mathrm{F})}+\Gamma_{\lambda'(\gamma\to\mathrm{II})}}+\frac{w_{\lambda''}(\Gamma_{\lambda''(\mathrm{F})}+b_{\mathrm{II}\to\mathrm{F}}\Gamma_{\lambda''(\gamma\to\mathrm{II})})}{w_{\lambda''}\Gamma_{\lambda_{\mathrm{I}}(\gamma\to\mathrm{I})}+\Gamma_{\lambda''(\mathrm{F})}+\Gamma_{\lambda''(\gamma\to\mathrm{II})}} \tag{17}$$

where the sum over λ' includes all quasi-class-I levels from $E_{\lambda_{\mathrm{II}}}-D_{\mathrm{II}}/2$ to $E_{\lambda_{\mathrm{II}}}+D_{\mathrm{II}}/2$. Equation (17) can be computed readily, the prescription

$$\overline{\Gamma}_{\lambda_{\mathrm{II}}(\mathrm{A})} = \frac{2\pi\overline{H^2_{\mathrm{c}}}}{D_{\mathrm{I}}} \tag{18}$$

being adopted.

A typical computation of this probability is given in figure 4. The radiation widths for both class-II and class-I states have been computed from the photonuclear giant resonance model (Brink 1955).

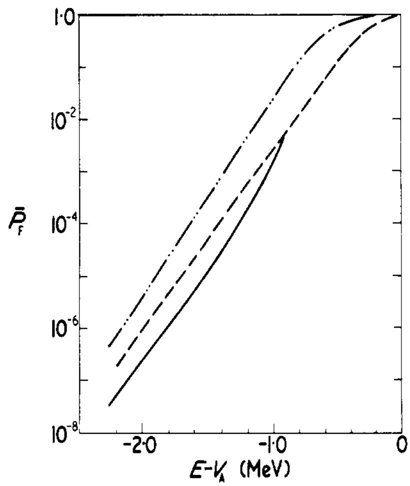

Figure 4. Fission probability calculated for the perturbation theory (full curve) (equation (17)). Parameters are $V_A = V_B = 5{\cdot}5$ MeV, $\hbar\omega_A = 0{\cdot}9$ MeV, $\hbar\omega_B = 0{\cdot}6$ MeV, E_{II} (ground state of secondary minimum) = 2·5 MeV. Radiation widths calculated from Brink giant resonance theory. The broken and chain curves are calculated from equations (8) and (5) respectively.

3. Examples of interpretation of data

3.1. Fission decay of even compound nuclei

Even (deformed) nuclei are characterized by the energy gap of the order of 1 MeV between the lowest rotational band associated with the ground state ($K^\pi = 0^+$) and the more complex states. This characteristic is expected to extend to the transition states associated with the barriers at greater deformations, with the exception that the outer barrier of the actinides is believed to have a mass asymmetric deformation which should allow the octupole vibration state ($K^\pi = 0^-$) and its associated rotational band ($I^\pi = 1^-, 3^-$ etc) to be nearly degenerate with the 0^+ band. Sub-barrier fission in even nuclei is expected to be dominated therefore by the fission decay of the compound nucleus states of spin and parity $0^+, 2^+, 4^+ \ldots$, with a minor contribution from the $1^-, 3^-, 5^- \ldots$ states.

As an example of the application of the analysis described in §2 we use the data on fission of ^{242}Pu through the ^{240}Pu(t, pf) reaction (figure 5). The relative weights for excitation of the states of different spin and parity in the (t, p) reaction have been calculated by Cramer and Britt (1970); they peak at 5 to 6 units of angular momentum. In the data on the ^{240}Pu(t, pf) reaction an inflection appears at about 4·6 MeV; this is interpreted as a damped vibrational resonance based on the 0^+ rotational band associated with the secondary well in the fission barrier. A model of the class-II structure has therefore been based on such a vibrational structure (the direct contribution T_D being neglected). In

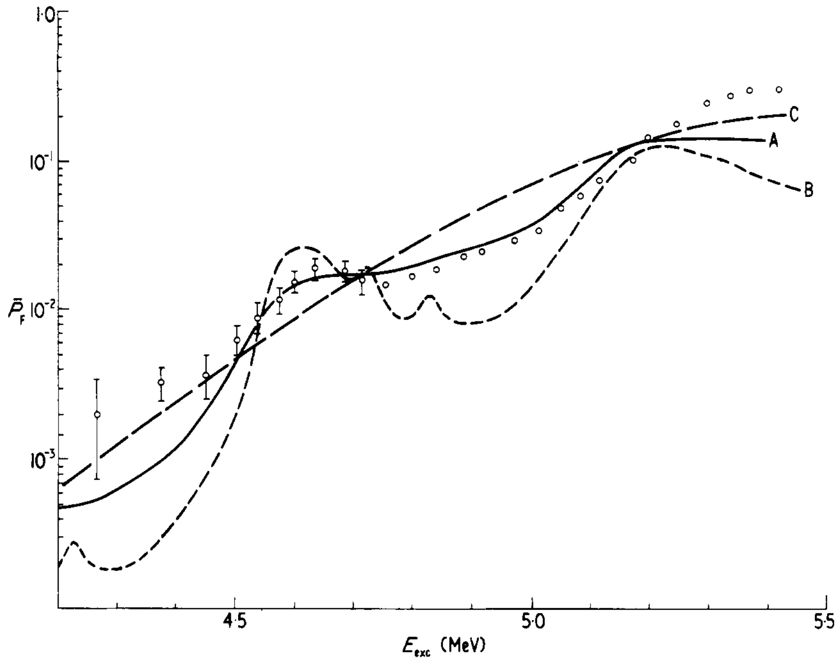

Figure 5. The fission probability in the reaction $^{240}Pu(t, pf)^{242}Pu$. Data are from Back and Britt (to be published). Calculated curves are for $V_A = 5{\cdot}6$ MeV, $\hbar\omega_A = 1{\cdot}0$ MeV, $V_B = 5{\cdot}1$ MeV, $\hbar\omega_B = 0{\cdot}7$ MeV. The damping half width for vibrational levels is 0·1 MeV for curve A and 0·025 MeV for the curve B. Curve C is calculated for strong damping. The effect of Porter–Thomas fluctuations in the class-I levels is not included; thus the barrier could be about 0.05 MeV lower than the calculation implies.

this model we have for the local average widths of class-II states (of one spin and parity)

$$\frac{\overline{\Gamma}_{\lambda_{\rm II}(B)}}{D_{\rm II}} = \frac{1}{2\pi}\sum_v \frac{2W_D\Gamma_{v(B)}}{(E_{\lambda_{\rm II}}-E_v)^2+(W_D+\frac{1}{2}\Gamma_{v(A)}+\frac{1}{2}\Gamma_{v(B)})^2} \tag{19}$$

$$\frac{\overline{\Gamma}_{\lambda_{\rm II}(A)}}{D_{\rm II}} = \frac{1}{2\pi}\sum_v \frac{2W_D\Gamma_{v(A)}}{(E_{\lambda_{\rm II}}-E_v)^2+(W_D+\frac{1}{2}\Gamma_{v(A)}+\frac{1}{2}\Gamma_{v(B)})^2} \tag{20}$$

where the subscript v denotes the vibrational resonances (separated, for each spin, by the vibrational energy in the second well, $\hbar\omega_{\rm II}$). The energies E_v, of the same value of v but different spin, are separated by the rotational energies for the spin sequence 0^+, 2^+ etc:

$$E_v(I^+) = E_v(0^+) + \frac{\hbar^2 I(I+1)}{2\mathscr{I}_{\rm II}}, \tag{21}$$

and the value 3·5 keV has been assumed for the rotational constant $\hbar^2/2\mathscr{I}_{\rm II}$. Similarly the barriers governing the widths $\Gamma_{v(A)}$, $\Gamma_{v(B)}$ (through equations (2) and (6)) are separated by rotational energies governed by rotational constants $\hbar^2/2\mathscr{I}_A$ (assumed 4 keV) and $\hbar^2/2\mathscr{I}_B$ (assumed 2·5 keV). Individual class-II widths are randomly selected from Porter–Thomas distributions with mean values given by (19, 20) and the results are used in equation (8). For any given energy the mean fission probability $\bar{P}_F$ is calculated from 100 trials with this procedure.

In the case of Pu nuclei it is believed that barrier B is about 0·5 MeV below barrier A. With this assumption it is found that a fit to the ^{240}Pu(t, pf) data can be achieved with $V_A = 5{\cdot}6$ MeV and barrier penetrability characteristics $\hbar\omega_A = 1{\cdot}0$ MeV, $\hbar\omega_B = 0{\cdot}7$ MeV. The radiation width for this fit has been calculated from Brink's (1955) photonuclear model; with $\Gamma_G = 4$ MeV, $E_G = 13$ MeV for the electric dipole giant resonance constants and the level density model of Gilbert and Cameron (1965) the radiation width of the resonances of the ^{241}Pu neutron cross section can be reproduced, and in the region of 5 to 6 MeV it is found that the radiation width can be represented by

$$\frac{\Gamma_\gamma(\mathrm{tot})}{D(J^\pi = 0^+)} = 2{\cdot}04 \times 10^{-9}\,\mathrm{e}^{E/0{\cdot}43} \tag{22}$$

3.2. Fission decay of odd-mass nuclei

The strong spin and parity selection rules that govern sub-barrier fission of an even compound nucleus are not expected to operate (except possibly in a very attenuated way) in the fission of odd-mass or odd nuclei. There is still expected to be an energy gap in the low-lying spectra of odd-mass nuclei, in which the level density does not show the strong exponential type of rise with energy characteristic of higher excitations, but this will be rather densely filled (especially at deformations corresponding to fission barrier peaks) with single quasi-particle states of varying spin and parity and their associated rotational bands. The available evidence suggests that the density of states in this low energy region is, for odd-mass actinide nuclei, perhaps an order of magnitude higher at barrier A than at normal deformation and at higher energies a factor of three to five higher. A very crude numerical representation (Lynn 1973) of the barrier A density has been suggested to be

$$\rho_A(E, J^\pi) = \rho_A(E)(2J+1)\exp\left(\frac{-(J+\frac{1}{2})^2}{2\sigma^2}\right) \tag{23}$$

with

$$\begin{aligned} \rho_A(E) &= 2{\cdot}0\,\mathrm{e}^{E/\tau_A} - 1{\cdot}25, && \text{if } 0 < E < 0{\cdot}5, \\ &= 2{\cdot}0\,\mathrm{e}^{E/\tau_A} + 3{\cdot}75 && \text{if } E > 0{\cdot}5 \end{aligned}$$

and $\sigma = 6{\cdot}1$, $\tau_A = 0{\cdot}39$. At barrier B it is believed that this density will be lower; for the purposes of the calculations here it is taken to be one half of the barrier A density. From these level density functions, barrier transmission functions can be computed; at sub-barrier energies a suitable approximation is

$$T_A(J^\pi) = (2J+1)\exp\left(\frac{-(J+\frac{1}{2})^2}{2\sigma^2}\right)\tau_A\rho_A(0)\frac{\hbar\omega_A}{2\pi\tau_A - \hbar\omega_A}\exp\left(\frac{2\pi(E-V_A)}{\hbar\omega_A}\right) \tag{24}$$

while above the barrier

$$\begin{aligned} T_A(J^\pi) = (2J+1)\exp\left(\frac{-(J+\frac{1}{2})^2}{2\sigma^2}\right)\tau_A\rho_A(0)\Bigg[\exp\left(\frac{E-V_A}{\tau_A}\right) - 1 \\ + \tfrac{1}{6}\pi^2\exp\left(\frac{E-V_A}{\tau_A}\right)\left(\frac{\hbar\omega_A}{2\pi\tau_A}\right)^2 + \dots\Bigg]. \end{aligned} \tag{25}$$

Equivalent formulae are used for T_B.

These equations have been applied to the analysis of the fission of ^{237}Np excited through the ^{236}U(^{3}He, d) reaction. Back and Britt have calculated the distribution function for excitation of the states of various angular momentum in the compound nucleus; it is peaked at $J \sim \frac{7}{2}$. The radiation transmission function is calculated from Brink's electric dipole giant resonance model and Gilbert and Cameron's recommendation for the level density parameter, to be

$$T_{\gamma}(J^{\pi}) = 4{\cdot}43 \times 10^{-8}\, e^{E/0{\cdot}43}(2J+1)\exp\left(-\frac{(J+\frac{1}{2})^2}{2\sigma^2}\right) \tag{26}$$

(σ here is for the excited compound nucleus and will be of the order of 6). The result for the fission probability, ignoring statistical fluctuation effects is shown in figure 6 for the choice of barrier parameters:

$$V_A = 5{\cdot}85 \text{ MeV}, \qquad \hbar\omega_A = 0{\cdot}8 \text{ MeV},$$

$$V_B = 5{\cdot}75 \text{ MeV}, \qquad \hbar\omega_B = 0{\cdot}8 \text{ MeV}.$$

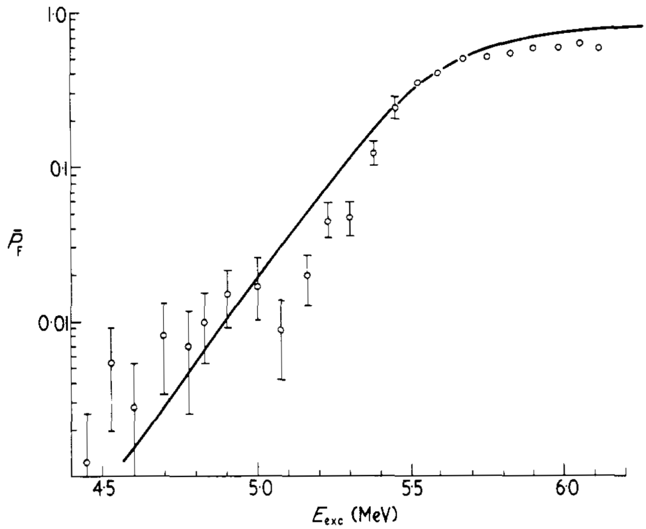

Figure 6. Fission probability in the reaction $^{236}U(^{3}He, df)^{237}Np$. Data are from Back and Britt (to be published).

Allowance for statistical fluctuations will probably lower these barriers by up to 0·1 MeV to retain the quality of fit. Equally good fits could be obtained if either barrier is raised one or two hundred keV while the other is lowered a corresponding amount.

3.3. Fission decay of doubly odd nuclei

Very similar considerations apply to doubly odd nuclei as to odd-mass nuclei. In this case however we assume that the barrier level densities are represented by

$$\begin{aligned} \rho_A &= 14(2J+1)\exp\left(-\frac{(J+\frac{1}{2})^2}{2\sigma^2}\right) e^{E/0{\cdot}36} \\ \rho_B &= 7(2J+1)\exp\left(-\frac{(J+\frac{1}{2})^2}{2\sigma^2}\right) e^{E/0{\cdot}36} \end{aligned} \tag{27}$$

Most of the odd actinide nuclei have neutron separation energies below the fission barrier. Thus, analysis of the fission probability data in the barrier region requires a knowledge of the competing neutron transmission function for these nuclei. A simple statistical expression has been developed for this (Lynn 1973); the result is shown in figure 7.

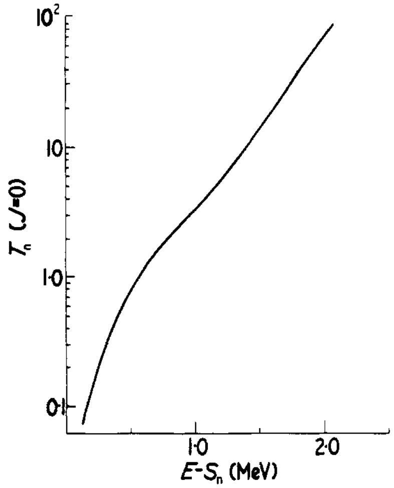

Figure 7. Neutron transmission coefficient for neutron emission from odd compound nuclei.

As illustration of the analysis of data on the fission of odd nuclei we take ^{238}Np, excited in the ^{237}Np(d, p) reaction. The angular momentum probability function peaks in the region of $J \sim 3$. The data and a number of attempted fits are shown in figure 8. The most satisfactory of these is based on the barrier parameters

$$V_A = V_B = 6{\cdot}18 \text{ MeV}$$

$$\hbar\omega_A = 0{\cdot}8 \text{ MeV}, \qquad \hbar\omega_B = 0{\cdot}6 \text{ MeV}.$$

With allowance for statistical fluctuation effects it is possible that barrier A may be almost 0·1 MeV higher while $\hbar\omega_B$ is probably closer to 0·7 MeV.

4. Discussion

It has been shown in this paper that fission decay of the compound nucleus can be critically dependent on the detailed intermediate structure due to the levels associated with the secondary well of the double-humped fission barrier. With due account of this structure estimates of fission barrier heights from analysis of sub-barrier fission data can be up to $\frac{1}{2}$ MeV lower than those obtained by simply averaging the fission widths over the intermediate structure.

The examples of such analysis given in § 3 reveal an important odd–even effect in barrier heights. Thus ^{242}Pu has been shown to have a barrier height of about 5·5 MeV, while the neighbouring nuclei ^{241}Pu and ^{243}Pu are known from the relevant fast neutron-induced fission cross sections to have barriers of approximately 5·8 MeV and 5·7 MeV

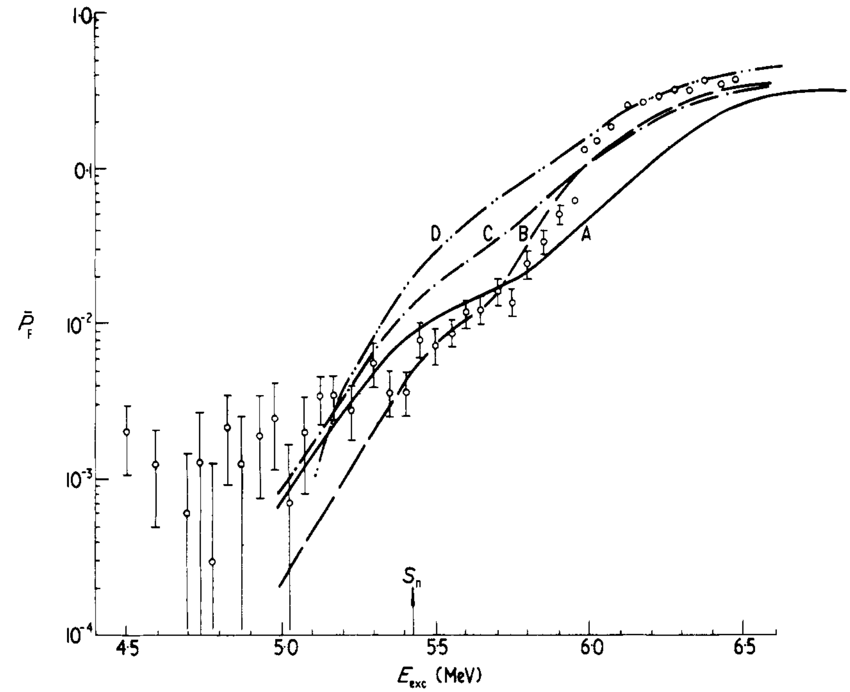

Figure 8. Fission probability in the reaction $^{237}Np(d, p)^{238}Np$. Data from Back *et al* (1971). Calculation parameters are:

curve A: $V_A = 6{\cdot}48$ MeV, $\hbar\omega_A = 0{\cdot}8$ MeV, $V_B = 5{\cdot}98$ MeV, $\hbar\omega_B = 0{\cdot}8$ MeV;
curve B: $V_A = 6{\cdot}18$ MeV, $\hbar\omega_A = 0{\cdot}8$ MeV, $V_B = 6{\cdot}18$ MeV, $\hbar\omega_B = 0{\cdot}6$ MeV;
curve C: $V_A = 6{\cdot}28$ MeV, $\hbar\omega_A = 0{\cdot}8$ MeV, $V_B = 6{\cdot}18$ MeV, $\hbar\omega_B = 0{\cdot}8$ MeV;
curve D: $V_A = 6{\cdot}28$ MeV, $\hbar\omega_A = 0{\cdot}8$ MeV, $V_B = 5{\cdot}88$ MeV, $\hbar\omega_B = 0{\cdot}6$ MeV.

respectively, ie about 0·2 to 0·3 MeV higher. Such a difference (but of magnitude closer to 0·4 MeV) is also apparent for the neptunium nuclei. This effect appears to be a general one, and had already been noticed by Back and Britt as likely qualitatively from an examination of their fission probability data; while it is also strongly indicated in the Am nuclei (Lynn 1973), for which ^{242}Am and ^{244}Am have barriers $V_A \simeq 6{\cdot}4$ MeV (from fast neutron-induced fission cross sections), and ^{243}Am obviously has a barrier much below its neutron separation energy (6·4 MeV), because ^{242}Am is extremely fissile to thermal and resonance region neutrons. These matters are more fully discussed in papers in preparation by Back and Britt, and Bjørnholm and Lynn.

Appendix. Derivation of the multilevel formula

Instead of a single class-II level, as used in the derivation of equation (8), we assume a spectrum of equidistant class-II states of uniform strength. This gives rise to a partial fission width of the form

$$\Gamma_f(E) = KD_I \sum_{n=-\infty}^{\infty} \frac{1}{(nD_{II}-X)^2+W^2} \tag{A.1}$$

where

$$K = \frac{\bar{T}_F D_{II} W}{2\pi^2} \tag{A.2}$$

and

$$X = E - E_0.$$

The half width of the class-II structures is denoted

$$W = \tfrac{1}{2}(\Gamma_{IIA} + \Gamma_{IIB}). \tag{A.3}$$

The sum in equation (1) can be split up in three parts, namely

$$\begin{aligned}
\frac{\Gamma_F}{KD_I} &= \frac{1}{X^2+W^2} + \sum_{n=1}^{\infty}\left(\frac{1}{(nD_{II}+X)^2+W^2} + \frac{1}{(nD_{II}-X)^2+W^2}\right) \\
&= \frac{1}{X^2+W^2} + \frac{1}{2iWD_{II}}\sum_{n=1}^{\infty}\left[\left(n+\frac{X}{D_{II}}-i\frac{W}{D_{II}}\right)^{-1} - \left(n+\frac{X}{D_{II}}+i\frac{W}{D_{II}}\right)^{-1}\right. \\
&\quad \left. + \left(n-\frac{X}{D_{II}}-i\frac{W}{D_{II}}\right)^{-1} - \left(n-\frac{X}{D_{II}}+i\frac{W}{D_{II}}\right)^{-1}\right].
\end{aligned} \tag{A.4}$$

Each of the four sums is a series expansion for the digamma function $\psi(Z)$, which has the form

$$\psi(1+b) = -\sum_{n=1}^{\infty}\frac{1}{n+b}. \tag{A.5}$$

Using the relation $\psi(\bar{Z}) = \overline{\psi(Z)}$ we find

$$\frac{\Gamma_F}{KD_I} = \frac{1}{X^2+W^2} + \frac{1}{2iD_{II}W}\left[2\,\mathrm{Im}\,\psi\left(1+\frac{X}{D_{II}}+i\frac{W}{D_{II}}\right) - 2\,\mathrm{Im}\,\psi\left(1-\frac{X}{D_{II}}+i\frac{W}{D_{II}}\right)\right]. \tag{A.6}$$

Employing the relations

$$\psi(1-Z) = \psi(Z) + \pi\cot(\pi Z) \qquad \text{and} \qquad \psi(1+Z) = \psi(Z) + \frac{1}{Z}$$

equation (6) reduces to

$$\frac{\Gamma_F}{D_I} = \frac{K}{X^2+W^2} + \frac{K}{iWD_{II}}\,\mathrm{Im}\left(\frac{D_{II}}{X+iW} - \pi\frac{\sin(2\pi X/D_{II}) - i\sinh(2\pi W/D_{II})}{\cosh(2\pi W/D_{II}) - \cos(2\pi X/D_{II})}\right) \tag{A.7}$$

$$= \frac{\bar{T}_F}{2\pi}\frac{\sinh(2\pi W/D_{II})}{\cosh(2\pi W/D_{II}) - \cos(2\pi X/D_{II})} \tag{A.8}$$

Averaged over a class-II resonance this is normalized to give the expected result $\bar{\Gamma}_F/\bar{D}_I = \bar{T}_F/2\pi$. The application of this formula to the calculation of fission probability in the general case that includes partial damping can be treated in a simple phenomenological way. For partial damping we assume that the fission transmission coefficient can be split up into two terms, one of which comes from direct penetration of the fission barrier, while the other comes from the part which is absorbed in the second well, takes part in the class-II compound motion and finally fissions by penetration of the second barrier. It is evident that only the second contribution contains the structure of equation (A.8). We therefore write the total transmission coefficient in the following way:

$$T_F = T_D + T_{Abs} \frac{T_B}{T_A + T_B} \frac{\sinh(2\pi W/D)}{\cosh(2\pi W/D) - \cos(2\pi X/D)}.$$

Here T_{Abs} is the probability for absorption in the second well.

The average fission probability is given by the integral

$$\bar{P}_F = \frac{1}{D_{II}} \int_{-D_{II}/2}^{D_{II}/2} \frac{T_F}{T_F + T'} \, dx$$

$$= \frac{1}{D_{II}} \int_{-D_{II}/2}^{D_{II}/2} \left(T_D + T_{Abs} \frac{T_B \sinh(2\pi W/D)}{(T_A + T_B)\left[\cosh(2\pi W/D) - \cos(2\pi X/D)\right]} \right)$$

$$\times \left(T_D + T_{Abs} \frac{T_B \sinh(2\pi W/D)}{(T_A + T_B)\left[\cosh(2\pi W/D) - \cos(2\pi X/D)\right]} + T' \right)^{-1} dx$$

$$= \frac{T_D}{T_D + T'} + \left\{ \left(1 + \frac{T_D}{T'}\right) \left[1 + \left(\frac{(T_D + T')(T_A + T_B)}{T_{Abs} T_B} \right)^2 \right. \right.$$

$$\left. \left. + 2 \frac{(T_D + T')(T_A + T_B)}{T_{Abs} T_B} \coth\left[\tfrac{1}{2}(T_A + T_B)\right] \right]^{1/2} \right\}^{-1}$$

where we have used

$$W = \frac{D_{II}}{2\pi} \frac{1}{2} (T_A + T_B).$$

References

Back B B, Bondorf J P, Otroschenko G A, Pedersen J and Rasmussen B 1971 *Nucl. Phys.* A **165** 449

Bjørnholm S and Strutinsky V M 1969 *Nucl. Phys.* A **136** 1

Bohr A 1956 *Proc. Int. Conf. Peaceful Uses of Atomic Energy, Geneva* 1955 vol 2 (New York: United Nations) p 220

Brink D M 1955 *D Phil. Thesis* Oxford University

Cramer J D and Britt H C 1970 *Nucl. Sci. Eng.* **41** 177

Cramer J D and Nix J R 1970 *Phys. Rev.* C **2** 1048

Flerov G N and Polikanov S M 1964 *C. R. Congr. Int. Physique Nucleaire, Paris* vol 1 p 407

Fubini A, Blons J, Michaudon A and Paya D 1968 *Phys. Rev. Lett.* **20** 1373

Gilbert A and Cameron A G W 1965 *Can. J. Phys.* **43** 1446

Lane A M and Lynn J E 1957 *Proc. Phys. Soc.* A **70** 557

Lark N L, Sletten G, Pedersen J and Bjørnholm S 1969 *Nucl. Phys.* A **139** 481

Lynn J E 1968 *Harwell Report* AERE-R5891

—— 1969 *Physics and Chemistry of Fission* (Vienna: IAEA) p 249

—— 1973 *Harwell Report* AERE-R7468

Migneco E and Theobald J P 1968 *Nucl. Phys.* A **112** 603

Porter C E and Thomas R G 1956 *Phys. Rev.* **104** 483

Strutinsky V M 1967 *Nucl. Phys.* A **95** 420

Weigmann H 1968 *Z. Phys.* **214** 7

Wong C Y and Bang J 1969 *Phys. Lett.* **29B** 143